Solution to Problem 88 of the 2001 Physics GRE

July 19, 2012

Recall the Biot-Savart Law, which states that

$$\frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{l}\times\hat{\mathbf{r}}}{r^2}$$

where $\mathbf{r}$ is the displacement vector from the element $d\mathbf{l}$ to the point where the magnetic field is being evaluated. In our case, the magnetic field at the point P is due only to the curved portions of the wire. Furthermore, in those curved portions, the vector $\mathbf{r}$ is always perpendicular to the vector $d\mathbf{l}$. Furthermore, the magnitude of the vector $\mathbf{r}$ is constant and always equal to R. So, the integral becomes

$$\frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{l}\times\hat{\mathbf{r}}}{r^2} = -\mathbf{z}\frac{\mu_0 I}{4\pi}\int_0^\theta \frac{1}{R^2}Rd\theta' = -\mathbf{z}\frac{\mu_0 I}{4\pi}\frac{\pi\theta}{R} = -\mathbf{z}\frac{\mu_0 I\theta}{4\pi R}$$

We are only asked for the magnitude of the magnetic field vector, therefore the answer is

$$\left|-\mathbf{z}\frac{\mu_0 I\theta}{4\pi R}\right| = \boxed{\frac{\mu_0 I\theta}{4\pi R}}$$

Therefore, the correct answer is (C).